Applications of Automata and Concurrency Theory in Networks

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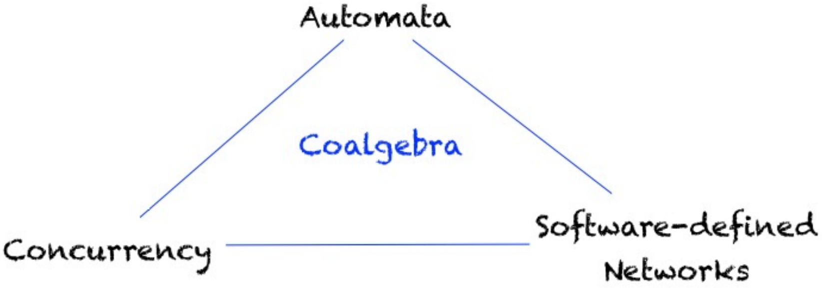
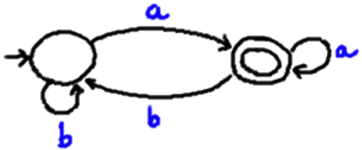
CONCUR 2015

Context


Automata
Coalgebra
Concurrency
Software-defined
Networks

Automata

- Automata are basic structures in Computer Science
- Language equivalence: well-studied, several algorithms



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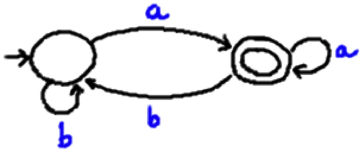
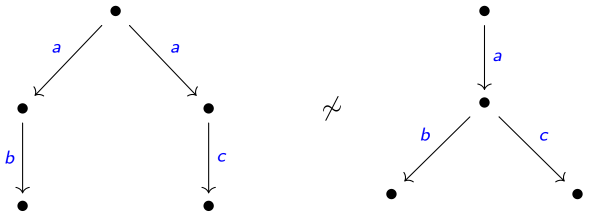


- New perspectives $\rightsquigarrow$ key algorithmic improvements

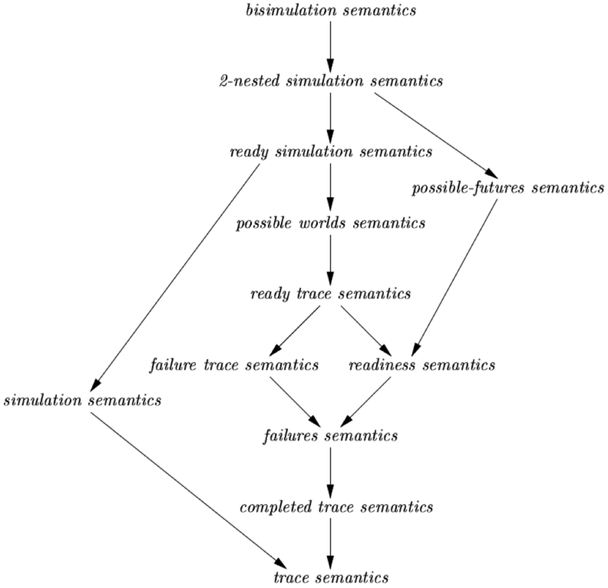
Concurrency

- Concurrency Theory: labelled transition systems



- Central research topic: a spectrum of equivalences

The spectrum of equivalences



Automata — Concurrency

- Many efficient algorithms for equivalence of automata.
- Applications in concurrency?

Automata —— Concurrency

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Various spectrum equivalences

=

Language equivalence of a *transformed* system

=

Automaton with outputs and structured states (Moore automaton).

Bonsangue, Bonchi, Caltais, Rutten, Silva. MFPS 12

Automata —— Concurrency

- Generalization of existing algorithms to Moore automata

Automata ——— Concurrency

- Generalization of existing algorithms to Moore automata
- Small step conceptually, great impact application-wise
- Method: coalgebra

Bonchi, Caltais, Pous, Silva. APLAS 2013

Algorithm derivation from the type

Equivalence/Minimization algorithms from the type $X \to TX$?

Deterministic automata	$X \to 2 \times X^A$
Moore automata	$X \to B \times X^A$
Linear weighted automata	$V \to \mathbb{R} \times V^A$
KAT automata	$\mathcal{B} \to \mathbb{B} \times \mathcal{B}^{\mathbb{B} \times A}$
$\vdots$	$\vdots$

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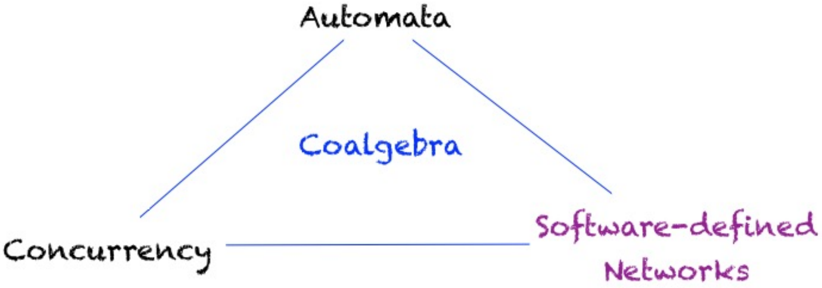

Automata
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Networking

"The last bastion of mainframe computing" [Hamilton 2009]

- Modern computers
  - implemented with commodity hardware
  - programmed using general-purpose languages, standard interfaces
- Networks
  - built and programmed the same way since the 1970s
  - low-level, special-purpose devices implemented on custom hardware
  - routers and switches that do little besides maintaining routing tables and forwarding packets
  - configured locally using proprietary interfaces
  - network configuration ("tuning") largely a black art

Networking

- Difficult to implement end-to-end routing policies and optimizations that require a global perspective
- Difficult to extend with new functionality
- Effectively impossible to reason precisely about behavior

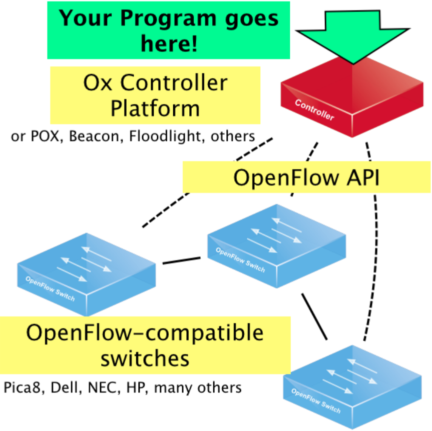
Software Defined Networks (SDN)

Main idea behind SDN

A general-purpose controller manages a collection of programmable switches

- controller can monitor and respond to network events
  - new connections from hosts
  - topology changes
  - shifts in traffic load
- controller can reprogram the switches on the fly
  - adjust routing tables
  - change packet filtering policies

SDN Network Architecture



Software Defined Networks (SDN)

Controller has a global view of the network

Enables a wide variety of applications:

- standard applications
  - shortest-path routing
  - traffic monitoring
  - access control
- more sophisticated applications
  - load balancing
  - intrusion detection
  - fault tolerance

Software Defined Networks (SDN)

> "*In the SDN architecture, the control and data planes are decoupled, network intelligence and state are logically centralized, and the underlying network infrastructure is abstracted from the applications. As a result, enterprises and carriers gain unprecedented programmability, automation, and network control, enabling them to build highly scalable, flexible networks that readily adapt to changing business needs.*"

—Open Networking Foundation, *Software-Defined Networking: The New Norm for Networks*, 2012

OpenFlow

A first step: the OpenFlow API [McKeown & al., SIGCOMM 08]

- specifies capabilities and behavior of switch hardware
- a language for manipulating network configurations
- very low-level: easy for hardware to implement, difficult for humans to write and reason about

But. . .

- is platform independent
- provides an open standard that any vendor can implement

A Major Trend in Industry

Backbone network

runs OpenFlow

Bought by VMware for $1.2B

Network Programming Languages & Analysis Tools

Goals:

- raise the level of abstraction above hardware-based APIs (OpenFlow)
- make it easier to build sophisticated and reliable SDN applications and reason about them

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- Formally Verifiable Networking [Wang & al., HotNets 09]
- FlowChecker [Al-Shaer & Saeed Al-Haj, SafeConfig 10]
- Anteater [Mai & al., SIGCOMM 11]
- Nettle [Voellmy & Hudak, PADL 11]
- Header Space Analysis [Kazemian & al., NSDI 12]
- Frenetic [Foster & al., ICFP 11] [Reitblatt & al., SIGCOMM 12]
- NetCore [Guha & al., PLDI 13] [Monsanto & al., POPL 12]
- Pyretic [Monsanto & al., NSDI 13]
- VeriFlow [Khurshid & al., NSDI 13]
- Participatory networking [Ferguson & al., SIGCOMM 13]
- Maple [Voellmy & al., SIGCOMM 13]

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NetKAT papers

Carolyn Jane Anderson, Nate Foster, Arjun Guha, Jean-Baptiste Jeannin, Dexter Kozen, Cole Schlesinger, and David Walker, NetKAT: Semantic Foundations for Networks. POPL 14.

Nate Foster, Dexter Kozen, Matthew Milano, Alexandra Silva, and Laure Thompson, A Coalgebraic Decision Procedure for NetKAT. POPL 15.

NetKAT

Simple programming language/logic, expressive enough for basic properties.

Reachability

- Can host A communicate with host B? Can every host communicate with every other host?

Security

- Does all untrusted traffic pass through the intrusion detection system located at C?
- Are non-SSH packets forwarded? Are SSH packets dropped?

Loop detection

- Is it possible for a packet to be forwarded around a cycle in the network?

Policy equivalence

- Given the network topology, are policies p and q equivalent?

NetKAT [Anderson & al. 14]

NetKAT

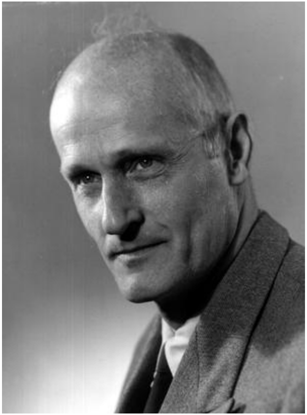
$=$

Kleene algebra with tests (KAT)

$+$

additional specialized constructs particular to network topology and packet switching

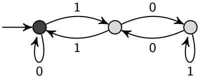
Kleene Algebra (KA)



Stephen Cole Kleene
(1909–1994)

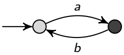
$(0 + 1(01^*0)^*1)^*$

{multiples of 3 in binary}



$(ab)^*a = a(ba)^*$

$\{a, aba, ababa, \ldots\}$



$(a + b)^* = a^*(ba^*)^*$

{all strings over $\{a, b\}$}

Foundations of the Algebraic Theory

John Horton Conway
(1937–)

J. H. Conway. *Regular Algebra and Finite Machines*. Chapman and Hall, London, 1971.

Axioms of KA

Idempotent Semiring Axioms

$$p + (q + r) = (p + q) + r \qquad p(qr) = (pq)r$$
$$p + q = q + p \qquad 1p = p1 = p$$
$$p + 0 = p \qquad p0 = 0p = 0$$
$$p + p = p$$
$$p(q + r) = pq + pr \qquad a \leq b \stackrel{\triangle}{\iff} a + b = b$$
$$(p + q)r = pr + qr$$

Axioms for *

$$1 + pp^* \leq p^* \qquad q + px \leq x \;\Rightarrow\; p^*q \leq x$$
$$1 + p^*p \leq p^* \qquad q + xp \leq x \;\Rightarrow\; qp^* \leq x$$

Standard Model

Regular sets of strings over Σ

$$
\begin{aligned}
A + B &= A \cup B \\
AB &= \{xy \mid x \in A,\ y \in B\} \\
A^* &= \bigcup_{n \geq 0} A^n \quad = \quad A^0 \cup A^1 \cup A^2 \cup \cdots \\
1 &= \{\varepsilon\} \\
0 &= \varnothing
\end{aligned}
$$

This is the free KA on generators Σ

Deciding KA

- PSPACE-complete [(1 + Stock)Meyer 74]
  - automata-based decision procedure
  - nondeterministically guess a string in $L(M_1) \oplus L(M_2)$, simulate the two automata
  - convert to deterministic using Savitch's theorem
  - inefficient—$\Omega(n^2)$ space, exponential time best-case
- coalgebraic decision procedures [Silva 10, Bonchi & Pous 12]
  - bisimulation-based
  - uses Brzozowski/Antimirov derivatives
  - Hopcroft–Karp union-find data structure, up-to techniques
  - implementation in OCaml
  - linear space, practical

Kleene Algebra with Tests (KAT)

$(K, B, +, \cdot, ^*, ^-, 0, 1), \quad B \subseteq K$

- $(K, +, \cdot, ^*, 0, 1)$ is a Kleene algebra
- $(B, +, \cdot, ^-, 0, 1)$ is a Boolean algebra
- $(B, +, \cdot, 0, 1)$ is a subalgebra of $(K, +, \cdot, 0, 1)$

- $p, q, r, \ldots$ range over K
- $a, b, c, \ldots$ range over B

Modeling While Programs

$$\begin{aligned} p; q &\triangleq pq \\ \textbf{if } b \textbf{ then } p \textbf{ else } q &\triangleq bp + \overline{b}q \\ \textbf{while } b \textbf{ do } p &\triangleq (bp)^*\overline{b} \end{aligned}$$

KAT Results

Deductive Completeness and Complexity

- deductively complete over language, relational, and trace models
- subsumes propositional Hoare logic (PHL)
- decidable in PSPACE

Applications

- protocol verification
- static analysis and abstract interpretation
- verification of compiler optimizations

NetKAT

NetKAT

- a packet π is an assignment of constant values n to fields x
- a packet history is a nonempty sequence of packets $\pi_1 :: \pi_2 :: \cdots :: \pi_k$
- the head packet is π_1

NetKAT

- assignments $x \leftarrow n$
  assign constant value n to field x in the head packet
- tests $x = n$
  if value of field x in the head packet is n, then pass, else drop
- dup
  duplicate the head packet

NetKAT

Example

$$sw = 6 \,;\, pt = 88 \,;\, dest \leftarrow 10.0.0.1 \,;\, pt \leftarrow 50$$

"For all packets incoming on port 88 of switch 6, set the destination IP address to 10.0.0.1 and send the packet out on port 50."

NetKAT Axioms

$$x \leftarrow n \,;\, y \leftarrow m \;\equiv\; y \leftarrow m \,;\, x \leftarrow n \quad (x \neq y)$$

assignments to distinct fields may be done in either order

$$x \leftarrow n \,;\, y = m \;\equiv\; y = m \,;\, x \leftarrow n \quad (x \neq y)$$

an assignment to a field does not affect a different field

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an assignment to a field does not affect a different field

$$x = n \,;\, \mathsf{dup} \;\equiv\; \mathsf{dup} \,;\, x = n$$

field values are preserved in a duplicated packet

$$x \leftarrow n \;\equiv\; x \leftarrow n \,;\, x = n$$

an assignment causes the field to have that value

$$x = n \,;\, x \leftarrow n \;\equiv\; x = n$$

an assignment of a value that the field already has is redundant

---

$$x \leftarrow n \,;\, x \leftarrow m \;\equiv\; x \leftarrow m$$

a second assignment to the same field overrides the first

$$x = n \,;\, x = m \;\equiv\; 0 \quad (n \neq m) \qquad\qquad (\textstyle\sum_n x = n) \;\equiv\; 1$$

every field has exactly one value

Standard Model

Standard model of NetKAT is a packet-forwarding model

$$[\![e]\!] : H \to 2^H$$

where $H = \{\text{packet histories}\}$

$$
\begin{aligned}
[\![x \leftarrow n]\!](\pi_1 :: \sigma) &\triangleq \{\pi_1[n/x] :: \sigma\} \\
[\![x = n]\!](\pi_1 :: \sigma) &\triangleq \begin{cases} \{\pi_1 :: \sigma\} & \text{if } \pi_1(x) = n \\ \varnothing & \text{if } \pi_1(x) \neq n \end{cases} \\
[\![\mathsf{dup}]\!](\pi_1 :: \sigma) &\triangleq \{\pi_1 :: \pi_1 :: \sigma\}
\end{aligned}
$$

Standard Model

$$
\begin{aligned}
[\![p + q]\!](\sigma) &\triangleq [\![p]\!](\sigma) \cup [\![q]\!](\sigma) \\
[\![p \,;\, q]\!](\sigma) &\triangleq \bigcup_{\tau \in [\![p]\!](\sigma)} [\![q]\!](\tau) \\
[\![p^*]\!](\sigma) &\triangleq \bigcup_n [\![p^n]\!](\sigma) \\
[\![1]\!](\sigma) &\triangleq [\![\text{pass}]\!](\sigma) \;=\; \{\sigma\} \\
[\![0]\!](\sigma) &\triangleq [\![\text{drop}]\!](\sigma) \;=\; \varnothing
\end{aligned}
$$

Example

Reachability

- Can host A communicate with host B? Can every host communicate with every other host?

Encoding Network Topology

Modeling Links

$$sw = A\,;\,pt = n\,;\,sw \leftarrow B\,;\,pt \leftarrow m$$

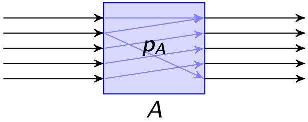
- filters out all packets not located at the source end of the link
- updates switch and port fields to the location of the target end
- this captures the effect of sending the packet across the link
- network topology is expressed as a sum of link expressions

Switch Policies

Switch behavior for switch A is specified by a NetKAT term

$$sw = A\,;\,p_A$$

where p_A specifies what to do with packets entering switch A



Example

$$pt = n\,;\,dest = a\,;\,dest \leftarrow b\,;\,(pt \leftarrow m + pt \leftarrow k)$$

Incoming packets on port n with destination $a \Rightarrow$ modify destination to b and send out on ports m and k

Switch policy p_A is the sum of all such behaviors for A

Putting It Together

Let

- t = sum of all link expressions
- p = sum of all switch policies

Then

- pt = one step of the network
- each switch processes its packets, then sends them along links to the next switch
- $(pt)^*$ = the multistep behavior of the network in which the single-step behavior is iterated

Reachability

To check if any packet can travel from A to B given the topology and the switch policies, ask whether

$$sw = A\,;\, t(pt)^*\,;\, sw = B \quad \not\equiv \quad 0\ (drop).$$

- prefix $sw = A$ filters out packets not at A
- suffix $sw = B$ filters out packets not at B

Other Applications

- forwarding loops
- traffic isolation
- access control
- correctness of a compiler that maps a NetKAT expression to a set of individual flow tables that can be deployed on the switches

Results

Soundness and Completeness [Anderson et al. 14]

- $\vdash p = q$ if and only if $[\![p]\!] = [\![q]\!]$

Decision Procedure [Foster et al. 15]

- NetKAT coalgebra
- Efficient bisimulation-based decision procedure
- Implementation in OCaml
- Deployed in the Frenetic suite of network management tools

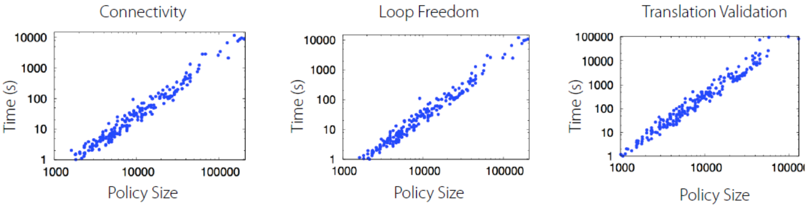
A Bisimulation-Based Algorithm

To check $e_1 = e_2$, convert to automata, check bisimilarity

- exploits a sparse matrix representation
- Hopcroft-Karp union-find data structure to represent bisimilarity classes
- BDDs to represent tests (new — based on Pous, POPL 15)
- algorithm is competitive with state of the art

A Bisimulation-Based Algorithm [Foster & al. 15]

- Topology Zoo
  - 261 real-world network topologies;
  - Use shortest path forwarding as network program;
  - Results:



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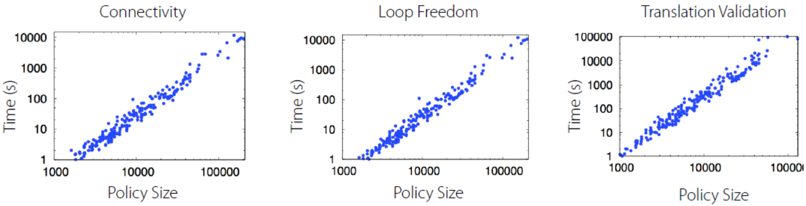


- Stanford Campus Network
  - Use actual router configurations
  - Results: Point to point reachability in 0.67s (vs 13s for HSA)

Probabilistic NetKAT

- How much congestion is there?
- Is the network resilient under failure?
- Reducing costs without compromising reliability

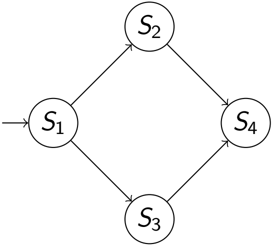
Probabilistic NetKAT

- How much congestion is there?
- Is the network resilient under failure?
- Reducing costs without compromising reliability

- Modular extension of NetKAT with probabilistic constructs
- Compositional semantics
- Compiler, Decision procedures, . . .

Compositional quantitative reasoning $\rightsquigarrow$ fully realize the vision of SDN

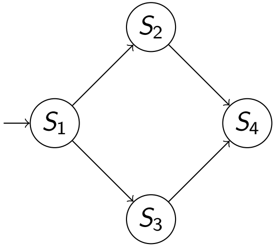
ProbNetKAT



10% probability of failure of the link $S_1 \to S_2$, topology t encoded as:

$$
\begin{aligned}
t = & (sw = S_1; pt = 2; ((sw \leftarrow S_2; pt \leftarrow 1) \oplus_{.9} \mathsf{drop})) \\
& \& \ (sw = S_1; pt = 3; sw \leftarrow S_3; pt \leftarrow 1) \\
& \& \ (sw = S_2; pt = 4; sw \leftarrow S_4; pt \leftarrow 2) \\
& \& \ (sw = S_3; pt = 4; sw \leftarrow S_4; pt \leftarrow 3).
\end{aligned}
$$

ProbNetKAT



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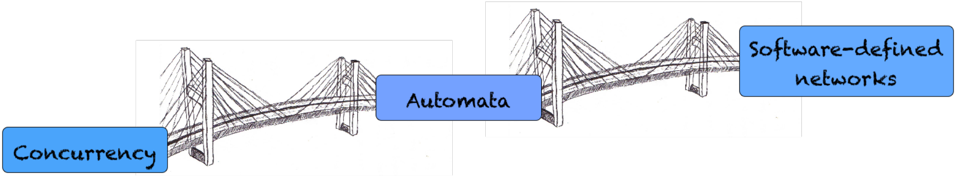
\end{aligned}
$$

Semantics in terms of Markov Kernels.

Conclusion

- Programming languages have a key role to play in emerging platforms for managing software-defined networks
- NetKAT is a high-level language for programming and reasoning about network behavior in the SDN paradigm
  - formal denotational semantics, complete deductive system
  - efficient bisimulation-based decision procedure
- Future work:
  - further optimizations to reduce state space
  - generating proof artifacts
  - refinement calculus
  - concurrent/distributed NetKAT
  - Many opportunities for the concurrency community!

Bridges



- Abstraction can bring new perspectives and solutions
- Transference of techniques is a two-way street
- Solid foundations are crucial for new paradigms

For papers and code, please visit:
http://frenetic-lang.org/

Thanks!